Lie superderivations of generalized matrix algebras

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Let $\mathcal{A}$ be an algebra.

We define Lie product $[x, y] := xy - yx$ and Jordan product $x \circ y := xy + yx$ for all $x, y \in \mathcal{A}$. Then $(\mathcal{A}, [,])$ becomes a Lie algebra and $(\mathcal{A}, \circ)$ is a Jordan algebra.

Superalgebra

An associative superalgebra, is a $\mathbb{Z}_2$-graded associative algebra. This means that there exist R-submodules $\mathcal{A}_0$ and $\mathcal{A}_1$ of $\mathcal{A}$ such that $\mathcal{A} = \mathcal{A}_0 \oplus \mathcal{A}_1$ and $\mathcal{A}_i\mathcal{A}_j \subseteq \mathcal{A}_{i+j}$, where indices are computed modulo 2.
Elements in $\mathcal{A}_0 \cup \mathcal{A}_1$, is said to be *homogeneous* of *degree* i and we write $|x| = i$ to mean $x \in \mathcal{A}_i$. We say that $\mathcal{A}_0$ is the *even* and $\mathcal{A}_1$ is the *odd* part of $\mathcal{A}$.

Superalgebra

Define a product in $\mathcal{A}_0 \cup \mathcal{A}_1$, the *supercommutator*, by

$$[x, y]_s = xy - (-1)^{|x||y|} yx$$

for $x, y \in \mathcal{A}$.

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for $x, y \in \mathcal{A}$. Note that in case $\mathcal{A} = \mathcal{A}_0$ the superproduct coincides with the Lie product. The *supercenter* of $\mathcal{A}$ is the set

$$\mathcal{Z}(\mathcal{A})_s = \{a \in \mathcal{A} \,|\, [a, x]_s = 0 \quad \text{for all} \;\; x \in \mathcal{A}\}.$$

Generalized matrix algebra

A **Morita context** consists of two unital R-algebras $\mathcal{A}$ and $\mathcal{B}$, two bimodules $(\mathcal{A}, \mathcal{B})$-bimodule $\mathcal{M}$ and $(\mathcal{B}, \mathcal{A})$-bimodule $\mathcal{N}$, and two bimodule homomorphisms called the bilinear pairings $\Phi_{\mathcal{M}\mathcal{N}} \colon \mathcal{M} \otimes_{\mathcal{B}} \mathcal{N} \to \mathcal{A}$ and $\Psi_{\mathcal{N}\mathcal{M}} \colon \mathcal{N} \otimes_{\mathcal{A}} \mathcal{M} \to \mathcal{B}$ satisfying the following commutative diagrams:

Generalized matrix algebra

$$\begin{array}{ccc} \mathcal{M}\otimes_{\mathcal{B}}\mathcal{N}\otimes_{\mathcal{A}}\mathcal{M} & \xrightarrow{\Phi_{\mathcal{M}\mathcal{N}}\otimes I_{\mathcal{M}}} & \mathcal{A}\otimes_{\mathcal{A}}\mathcal{M} \\ {\scriptstyle I_{\mathcal{M}}\otimes\Psi_{\mathcal{N}\mathcal{M}}}\Big\downarrow & & \Big\downarrow{\scriptstyle\cong} \\ \mathcal{M}\otimes_{\mathcal{B}}\mathcal{B} & \xrightarrow[\cong]{\qquad\qquad} & \mathcal{M} \end{array}$$

and

$$\begin{array}{ccc} \mathcal{N}\otimes_{\mathcal{A}}\mathcal{M}\otimes_{\mathcal{B}}\mathcal{N} & \xrightarrow{\Psi_{\mathcal{N}\mathcal{M}}\otimes I_{\mathcal{N}}} & \mathcal{B}\otimes_{\mathcal{B}}\mathcal{N} \\ {\scriptstyle I_{\mathcal{N}}\otimes\Phi_{\mathcal{M}\mathcal{N}}}\Big\downarrow & & \Big\downarrow{\scriptstyle\cong} \\ \mathcal{N}\otimes_{\mathcal{A}}\mathcal{A} & \xrightarrow[\cong]{\qquad\qquad} & \mathcal{N}. \end{array}$$

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We write this Morita context as $(\mathcal{A}, \mathcal{B}, \mathcal{M}, \mathcal{N}, \Phi_{\mathcal{MN}}, \Psi_{\mathcal{NM}})$

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then the set

$$\mathcal{G} = \begin{bmatrix} \mathcal{A} & \mathcal{M} \\ \mathcal{N} & \mathcal{B} \end{bmatrix} = \left\{ \begin{bmatrix} a & m \\ n & b \end{bmatrix} \mid a \in \mathcal{A}, m \in \mathcal{M}, n \in \mathcal{N}, b \in \mathcal{B} \right\}$$

forms an R-algebra under matrix opreations, where at least one of the two bimodules $\mathcal{M}$ and $\mathcal{N}$ is distinct from zero. Such an R-algebra is usually called a *generalized matrix algebra*.

Generalized matrix algebra

By letting

$$\mathcal{G}_0 = \begin{pmatrix} \mathcal{A} & \\ & \mathcal{B} \end{pmatrix}, \mathcal{G}_1 = \begin{pmatrix} & \mathcal{M} \\ \mathcal{N} & \end{pmatrix}$$

It is easily verified that

$$\mathcal{G} = \begin{pmatrix} \mathcal{A} & \mathcal{M} \\ \mathcal{N} & \mathcal{B} \end{pmatrix}$$

is a superalgebra and

$$\mathcal{Z}(\mathcal{G})_s = \mathcal{Z}(\mathcal{G}).$$

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is a superalgebra and

$$\mathcal{Z}(\mathcal{G})_s = \mathcal{Z}(\mathcal{G}).$$

Note that

$$\mathcal{Z}(\mathcal{G}) = \{a \oplus b \,|\, a \in \mathcal{A}, b \in \mathcal{B}, am = mb, na = bn, \text{ for all } m \in \mathcal{M}, n \in \mathcal{N}\}$$

where

$$a \oplus b = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}.$$

It is a fascinating topic to study the connection between the associative, Lie and Jordan structures on $\mathcal{A}$. In this field, two classes of mappings are of crucial importance.

Differential operators

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The other one is formed by differential operators, satisfying a type of Leibniz formulas, such as Lie derivations and Jordan derivations.

Differential operators

R-linear map d from $\mathcal{A}$ into an $\mathcal{A}$-bimodule $\mathcal{M}$ is a *derivation* if

$$d(xy) = d(x)y + xd(y)$$

for all $x, y \in \mathcal{A}$.

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It is called a *Lie derivation* if

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$$d([x, y]) = [d(x), y] + [x, d(y)]$$

for all $x, y \in \mathcal{A}$.

Clearly, each derivation is a Lie derivation, but the converse is not true in general.

Differential operators

A standard example of a Lie derivation is of the form $d = \delta + \tau$, where $\delta\colon \mathcal{A} \to \mathcal{A}$ is a derivation and $\tau\colon \mathcal{A} \to \mathcal{Z}(\mathcal{A})$ is a linear map, where $\mathcal{Z}(\mathcal{A})$ denotes the center of $\mathcal{A}$, such that $\tau([a, b]) = 0$, for all $a, b \in \mathcal{A}$.

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Therefore, the Lie derivation $d\colon \mathcal{A} \to \mathcal{A}$ is in *standard form* iff $d = \delta + \tau$, where δ is a derivation of $\mathcal{A}$ and τ is a linear center valued map on $\mathcal{A}$ and vanishes at commutators. There are many papers concerning the study of conditions, which Lie derivations of rings or algebras are in standard form.

Over the recent years there has been a considerable interest in the study of superalgebra versions of Herstein's theorem on superderivations.

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Motivated by [3] (2-local superderivations on a superalgebra $M_n(\mathbb{C})$) and [1] (On superderivations and super-biderivations of trivial extensions and triangular matrix rings), in this paper we will address the structure of Lie superderivations on generalized matrix algebra.

As you see in below, we naturally obtain the description concerning superderivations of $\mathcal{G}$ via the characterization of derivations and Lie superderivations on $\mathcal{G}$.

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The purpose is to identify a class of superalgebras for which every Lie superderivation is in standard form.

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$d_i(xy) = d_i(x)y + (-1)^{i|x|} x d_i(y)$ for all $x, y \in A_0 \cup A_1$.

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$d_i(xy) = d_i(x)y + (-1)^{i|x|} x d_i(y)$ for all $x, y \in A_0 \cup A_1$.

A superderivation of $\mathcal{A}$ is the sum of a superderivation of degree 0 and a superderivation of degree 1. Note that every superderivation of degree 0 is actually a derivation from $\mathcal{A}$ to $\mathcal{A}$.

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$x, y \in A_0 \cup A_1$.

A Lie superderivation is the sum of a Lie superderivation of degree 0 and a Lie superderivation of degree 1.

In the case of trivial superalgebras (i.e., the odd part is 0) the consept of a Lie superderivation coincides with that of a Lie derivation.

From now on, we assume that the modules $\mathcal{M}$ and $\mathcal{N}$ appeared in the generalized matrix algebra $\mathcal{G}$ are *2-torion free* ($\mathcal{M}$ is called 2-torsion free if $2m = 0$ implies $m = 0$ for any $m \in \mathcal{M}$).

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$$d_0 \begin{pmatrix} a & m \\ n & b \end{pmatrix} = \begin{pmatrix} \alpha_1(a) + \alpha_4(b) & \mu_2(m) \\ \nu_3(n) & \beta_1(a) + \beta_4(b) \end{pmatrix}$$

where $\alpha_1 \colon \mathcal{A} \to \mathcal{A}$, $\alpha_4 \colon \mathcal{B} \to Z(\mathcal{A})$, $\mu_2 \colon \mathcal{M} \to \mathcal{M}$, $\nu_3 \colon \mathcal{N} \to \mathcal{N}$, $\beta_1 \colon \mathcal{A} \to \mathcal{B}$, $\beta_4 \colon \mathcal{B} \to Z(\mathcal{B})$ are linear maps satisfying the following conditions

1. α_1, β_4 *are Lie derivations;*
2. $\beta_1([a, a']) = 0$ *for all* $a, a' \in \mathcal{A}$ *and* $\alpha_4([b, b']) = 0$ *for all* $b, b' \in \mathcal{B}$*;*
3. $\mu_2(am) = a\mu_2(m) + \alpha_1(a)m - m\beta_1(a)$*;*
4. $\mu_2(mb) = \mu_2(m)b + m\beta_4(b) - \alpha_4(b)m$*;*
5. $\nu_3(na) = n\alpha_1(a) + \nu_3(n)a - \beta_1(a)n$*;*
6. $\nu_3(bn) = \beta_4(b)n + b\nu_3(n) - n\alpha_4(b)n$*;*
7. $\alpha_1(mn) = \mu_2(m)n + m\nu_3(n) - \alpha_4(nm)$*;*
8. $\beta_4(nm) = n\mu_2(m) + \nu_3(n)m - \beta_1(mn)$*.*

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for some $m_0 \in \mathcal{M}, n_0 \in \mathcal{N}$.

Theorem

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wher $m_0 \in \mathcal{M}, n_0 \in \mathcal{N}$ and $\alpha_1 : \mathcal{A} \to \mathcal{A}$, $\alpha_4 : \mathcal{B} \to Z(\mathcal{A})$, $\mu_2 : \mathcal{M} \to \mathcal{M}$, $\nu_3 : \mathcal{N} \to \mathcal{N}$, $\beta_1 : \mathcal{A} \to \mathcal{B}$, $\beta_4 : \mathcal{B} \to Z(\mathcal{B})$ are linear maps satisfying the following conditions:

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wher $m_0 \in \mathcal{M}, n_0 \in \mathcal{N}$ and $\alpha_1 \colon \mathcal{A} \to \mathcal{A}$, $\alpha_4 \colon \mathcal{B} \to Z(\mathcal{A})$, $\mu_2 \colon \mathcal{M} \to \mathcal{M}$, $\nu_3 \colon \mathcal{N} \to \mathcal{N}$, $\beta_1 \colon \mathcal{A} \to \mathcal{B}$, $\beta_4 \colon \mathcal{B} \to Z(\mathcal{B})$ are linear maps satisfying the following conditions:

1. α_1, β_4 *are Lie derivations;*
2. $\beta_1([a, a']) = 0$ *for all* $a, a' \in \mathcal{A}$ *and* $\alpha_4([b, b']) = 0$ *for all* $b, b' \in \mathcal{B}$;
3. $\mu_2(am) = a\mu_2(m) + \alpha_1(a)m - m\beta_1(a)$;
4. $\mu_2(mb) = \mu_2(m)b + m\beta_4(b) - \alpha_4(b)m$;
5. $\nu_3(na) = n\alpha_1(a) + \nu_3(n)a - \beta_1(a)n$;
6. $\nu_3(bn) = \beta_4(b)n + b\nu_3(n) - n\alpha_4(b)n$;
7. $\alpha_1(mn) = \mu_2(m)n + m\nu_3(n) - \alpha_4(nm)$;
8. $\beta_4(nm) = n\mu_2(m) + \nu_3(n)m - \beta_1(mn)$.

In order to proceed with our work to identify sufficient conditions on generalized matrix algebra $\mathcal{G}$ as a class of superalgebras for which every Lie superderivations is written in the standard form we are forced to characterize superderivations of generalized matrix algebra and supercentral mapping of $\mathcal{G}$ which maps supercommutators to 0.

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Since every superderivation of degree 0 on a superalgebra $\mathcal{A}$ is actually a derivation with the property $d_0(\mathcal{A}_0) \subseteq \mathcal{A}_0$ and $d_0(\mathcal{A}_1) \subseteq \mathcal{A}_1$, and by concerning derivations of $\mathcal{G}$ in [5, Propositions 4.2], we get:

Proposition

Let $\delta_0 \colon \mathcal{G} \to \mathcal{G}$ be a linear map. Then δ_0 is a superderivation degree 0 if and only if δ_0 has the form

$$\delta_0 \begin{pmatrix} a & m \\ n & b \end{pmatrix} = \begin{pmatrix} \alpha_1(a) & \mu_2(m) \\ \nu_3(n) & \beta_4(b) \end{pmatrix}$$

where $\alpha_1 \colon \mathcal{A} \to \mathcal{A}$, $\mu_2 \colon \mathcal{M} \to \mathcal{M}$, $\nu_3 \colon \mathcal{N} \to \mathcal{N}$, $\beta_1 \colon \mathcal{B} \to \mathcal{B}$ are linear maps satisfying the following conditions:

1. α_1, β_4 *are derivations;*
2. $\mu_2(am) = a\mu_2(m) + \alpha_1(a)m$, $\mu_2(mb) = \mu_2(m)b + m\beta_4(b)$;
3. $\nu_3(na) = n\alpha_1(a) + \nu_3(n)a$, $\nu_3(bn) = \beta_4(b)n + b\nu_3(n)$;
4. $\alpha_1(mn) = \mu_2(m)n + m\nu_3(n)$, $\beta_4(nm) = n\mu_2(m) + \nu_3(n)m$.

Proposition

Let $\delta_1 : \mathcal{G} \to \mathcal{G}$ *be a linear map. Then* δ_1 *is a superderivation degree 1 if and only if* δ_1 *has the form*

$$\delta_1 \begin{pmatrix} a & m \\ n & b \end{pmatrix} = \begin{pmatrix} mn_0 - m_o n & am_0 - m_0 b \\ n_0 a - bn_0 & n_0 m - nm_0 \end{pmatrix}$$

for some $m_0 \in \mathcal{M}, n_0 \in \mathcal{N}$.

Corollary

A superderivation δ on $\mathcal{G}$ is of the form

$$\delta\begin{pmatrix} a & m \\ n & b \end{pmatrix} = \begin{pmatrix} \alpha_1(a) + mn_0 - m_o n & am_0 - m_0 b + \mu_2(m) \\ n_0 a - bn_0 + \nu_3(n) & n_0 m - nm_0 + \beta_4(b) \end{pmatrix}$$

for some $m_0 \in \mathcal{M}, n_0 \in \mathcal{N}$ and linear maps $\alpha_1 : \mathcal{A} \to \mathcal{A}$, $\mu_2 : \mathcal{M} \to \mathcal{M}$, $\nu_3 : \mathcal{N} \to \mathcal{N}$, $\beta_1 : \mathcal{B} \to \mathcal{B}$, satisfying the following conditions:

1. α_1, β_4 *are derivations;*
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4. $\alpha_1(mn) = \mu_2(m)n + m\nu_3(n)$, $\beta_4(nm) = n\mu_2(m) + \nu_3(n)m$.

Proposition

Let $\mathcal{G}$ be a generalized matrix algebra. A linear mapping τ is supercenter valued and vanishes at supercommutators if and only if τ has the presentation

$$\tau\begin{pmatrix} a & m \\ n & b \end{pmatrix} = \begin{pmatrix} \gamma_1(a)+\gamma_4(b) & \\ & \lambda_1(a)+\lambda_4(b) \end{pmatrix}$$

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where $\gamma_1 : \mathcal{A} \to Z(\mathcal{A})$, $\gamma_4 : \mathcal{B} \to Z(\mathcal{A})$, $\lambda_1 : \mathcal{A} \to Z(\mathcal{B})$, $\lambda_4 : \mathcal{B} \to Z(\mathcal{B})$ are linear maps vanishing at commutators, having the following properties:

1. $\gamma_1(a) \oplus \lambda_1(a) \in Z(\mathcal{G})$ *and* $\gamma_4(b) \oplus \lambda_4(b) \in Z(\mathcal{G})$;
2. $\gamma_1(mn) = -\gamma_4(nm)$ *and* $\lambda_1(mn) = -\lambda_4(nm)$.

Following the method of [2, Theorem 6], the next theorem states a necessary and suffcient condition for a Lie superderivation on a general matrix algebra to be in stadard form.

Theorem

Let $\mathcal{G}$ be a generalized matrix algebra. A Lie superderivation d on $\mathcal{G}$ of the form

$$d\begin{pmatrix} a & m \\ n & b \end{pmatrix} = \begin{pmatrix} \alpha_1(a) + mn_0 - m_0 n + \alpha_4(b) & am_0 - m_0 b + \mu_2(m) \\ n_0 a - bn_0 + \nu_3(n) & \beta_1(a) + n_0 m - nm_0 + \beta_4(b) \end{pmatrix}$$

is in standard form if and only if there exist linear mappings $\gamma_{\mathcal{A}}\colon \mathcal{A} \to Z(\mathcal{A})$ and $\gamma_{\mathcal{B}}\colon \mathcal{B} \to Z(\mathcal{B})$ satisfying:

1. $\alpha_1 - \gamma_{\mathcal{A}}$ *is a derivation on* $\mathcal{A}$ *and* $\beta_4 - \gamma_{\mathcal{B}}$ *is a derivation on* $\mathcal{B}$*;*
2. $\gamma_{\mathcal{A}}(a) \oplus \beta_1(a) \in Z(\mathcal{G})$ *and* $\alpha_4(b) \oplus \gamma_{\mathcal{B}}(b) \in Z(\mathcal{G})$*;*
3. $\gamma_{\mathcal{A}}(mn) = -\alpha_4(nm)$ *and* $\beta_1(mn) = -\gamma_{\mathcal{B}}(nm)$*.*

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Thank you for attending!